**Problem 17-3:** A rod of mass m, length h and rotational inertia I sits on a frictionless table. It can pivot about a frictionless pin at one end. Point D identifing the center (midpoint) of the rod. Assume the rod starts out at rest. A disk of mass  $m_d$  slides toward the rod with velocity  $v_o$  perpendicular to the

rod, sticking to the rod a distance x units from the



pivot. The student wants the rod-disk system to end up with as much angular velocity as possible.

a.) If the rod is much more massive than the disk, should the disk strike the rod *to the left of D*, *directly at the center (at D)* or *to the right of D*? Briefly explain your reasoning without manipulating equations.

--angular momentum will be conserved if there are no external, torque-related impulses acting about the pin, which is the case here;

--the disk's angular momentum just before it hits the rod will be a function of its mass, its speed and how far out along the rod it is (mvx);

--the greater the systems angular momentum, the greater the rod/disk's after-collision angular velocity (L=Iw, where "w" is defined as angular velocity);

--therefore, the disk should hit to the right of the Point C so that x will be as large as possible

b.) A student finds the following equation for the post-collision angular velocity of the rod to

be  $\omega = \frac{m_d x v_o}{I}$ . Ignoring whether this equation is correct, does it agree with your

conclusion in Part a? In other words, does this equation have the expected dependence as reasoned in Part a? Briefly explain your reasoning without deriving an equation for  $\omega$ .

--this relationship suggests that the angular velocity will be directly related to the distance x from the pin, which is what my analysis above stated.

--the analysis suggest that the angular velocity will be a function of the velocity of the incoming disk, which my analysis also suggested

c.) Another students derives the post-collision angular velocity as  $\omega = \frac{lxv_o}{m_d h^4}$ . This is

incorrect. Without doing the derivation, how can you tell that this is not plausible? That is, how can you tell it doesn't make sense? Briefly explain.

--ignoring dimensional analysis, this relationship suggests that the smaller the incoming disk mass, the greater the final angular velocity—this makes no sense;

--this relationship also suggests that the larger the rod's moment of inertia I (being a rotational inertia term), the greater the final angular velocity . . . which also doesn't make any sense

For Parts d and e, do NOT assume that the rod is much more massive than the disk.

d.) Immediately before the collision, the disk's rotational inertia about the pivot will be  $I_{init,d} = m_d x^2$  and its angular momentum will be  $m_d v_o x$ . Explain how you would determine the rod's angular momentum after the collision.

--angular momentum is conserved in this collision;

--the rod's initial angular momentum is zero (it starts from rest);

--the disk's initial angular momentum is mvx;

--the disk exerts a torque on the rod angularly accelerating it to its "final" angular velocity, while the rod exerts and opposite torque on the disk angularly accelerating it to that same "final" angular velocity (the two bodies stay together);

--the final angular momentum is the sum of the moment of inertias  $(I + mx^2)$  times the new angular velocity;

--the rod's angular momentum will be that angular velocity times its moment of inertia, or Iw.

e.) Reconsider the collision alluded to in Part d, but this time assume the disk doesn't stick to the rod but, instead, bounces back away from the rod. Is the post-collision angular speed of the rod *greater than*, the *same* as or *less than* the angular speed of the rod in the situation outlined in Part d? Briefly explain your reasoning without the use of equations.

Alternate 1:

--upon collision, the disk applied a torque to the rod, angularly accelerating it;

--the rod applied an equal and opposite torque on the disk motivating it to alter its angular velocity;

--when the disk stuck to the rod, all the torque on the disk had to do was to slow the disk so that its angular velocity match that of the rod;

--but if the disk were to bounce back, the torque would have to do that PLUS motivate the disk to *bounce back* in the opposite direction, which means the torque would have to be EVEN LARGER than it had been in the original situation;

--with a larger torque being applied to the disk, and hence rod, the rod's "final" angular speed must be greater than it would otherwise have been.

Alternate 2:

--because there are no external, torque-like impulses being applied about the pin, *angular momentum* is conserved through the collision;

--in the original scenario, both the rod and disk had the same final angular velocity w, so their angular momentum was "Iw +  $(mx^2)w$ ."

--with bounce back, now that same value has to accommodate a *negative* angular momentum for the disk, which means the angular momentum for the rod has to be *larger* than before;

--a larger angular momentum for the rod means a greater "final" angular velocity for the rod than would otherwise be the case.